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Invariant Contra Harmonic Mean Graph Labeling

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Abstract: *A Graph G is an Invariant contra harmonic graph(ICHMG), for a 1-1 mapping $z: V \to 1,2,3,\dots,p$, there exist an induced mapping $z^*: E(G)$ to N which is given by $z^*(ab) = \left\lceil \frac{z(a)z(b)\{z(a)+z(b)\}}{z^2(a)+z^2(b)} \right\rceil$ or $z^*(ab) = \left\lfloor \frac{z(a)z(b)\{z(a)+z(b)\}}{z^2(a)+z^2(b)} \right\rfloor$ for all distinct $ab \in E(G)$. This paper provides ICHMG of path, Comb, Ladder L_n, Square graph P_2 and Broom graphs are discussed.*

Keywords *- Invariant contra harmonic mean, Path, Comb, ladder, Square graph*

I. Introduction

Greek means and their brief collection is discussed by Georghe Toader and Silvia Toader [1]. Also, comparison of their properties, partial derivatives of means and related results. Somasundaram and Ponraj were introduced the labeling of graphs and its notions [14, 15]. Gallian[2] gave the detailed survey on Graph labeling and its different applications. Results on labeling were found [2, 3] and mean labeling in ([4]-[15]).

Definition: Any graph G stated as a pair of V & E, here V represent non-empty set and E represent set of un-ordered pairs of elements of V.

Definition: A simple graph is one that lacks loops and multiple edges.

Definition: A finite graph is a graph with a finite number of vertices and edges is undirected and has p vertices and q edges.

Definition: The cardinality of a graph is set of vertices V is termed the order, and the cardinality of its edge set E is called the size of graph G. By removing an edge e from G, the graph G - e is obtained.

Definition: When $2 \geq d(u,v)$ in the graph G, then G^2 indicate - square of G has $V(G^2)$ with u, v adjacent in G^2. The results of a comprehensive survey on graph labelling may be found in [3].

Definition: For $a, b > 0$, the contra harmonic mean and the invariant contra harmonic mean are respectively given by $Z(a,b) = \frac{a^2+b^2}{a+b}$. and $Z^*(a,b) = \frac{ab(a+b)}{a^2+b^2}$.

Definition: The term "graph" G with n-vertices and m-edges is referred an Invariant Contra Harmonic mean graph. If $f: V(G) \to A \subseteq N$ is utilized to label the vertices x belongs to V(G) with separate f(x) labels and each edge $e = \{x_i\, x_j\}$ is labeled as $f^*(xy) = \left\lceil \frac{z(x)z(y)\{z(x)+z(y)\}}{(f(x))^2+(f(y))^2} \right\rceil$ or $f^*(xy) = \left\lfloor \frac{f(x)f(y)\{f(x)+f(y)\}}{(f(x))^2+(f(y))^2} \right\rfloor$ for every $x_i, x_j \in V(G)$ and $x_i \neq x_j$ are all distinct.

II. Main Results

Theorem 2.1. Path is an invariant contra harmonic graph labeling.

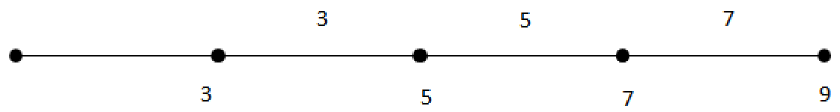
Proof: A path P_n with q edges, if z is a function from the vertices of G to N and z^* is injective mapping defined as z^*:E(G) to N defined by;

$z^*(xy) = \left\lceil \frac{z(x)z(y)\{z(x)+z(y)\}}{z^2(x)+z^2(y)} \right\rceil$ or $z^*(xy) = \left\lfloor \frac{z(x)z(y)\{z(x)+z(y)\}}{z^2(x)+z^2(y)} \right\rfloor$

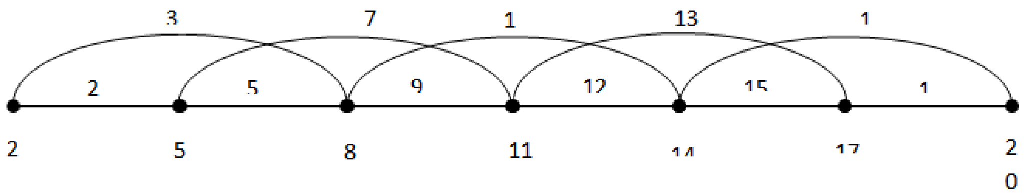
For every $x_i, x_j \in V(G)$ and $x_i \neq x_j$.

The edges that emerge are distinct..

As a result, the path is an invariant contra harmonic labeling graph.
It's worth noting that if the vertices are labeled with odd, even and natural numbers the edges admits odd, even and natural numbers labeling, with invariant contra harmonic graph labeling.
Illustration: The Invariant contra harmonic mean labeling of path with 5 vertices are as shown in the following figure.



Theorem 2.2. Square graph is an invariant contra harmonic graph labeling.
Proof: If P_n has n vertices $x_1, x_2, x_3 \ldots., x_n$ and when ever $d(u,v) \le 2$, then P_n^2 has n vertices and $(2n-3)$ edges.
Define $z: V(P_n^2) \to N$ such that $z(x_j) = 2j-1, 1 \le j \le n$, then
$z^*(x_i x_{i+1}) = \left\lfloor \frac{z(x_i)z(x_{i+1})\{z(x_i)+z(x_{i+1})\}}{z^2(x_i)+z^2(x_{i+1})} \right\rfloor$,
for every $\{x_i, x_{i+1}\} = e \in E(P_n^2)$
The edges $\{x_i x_{i+2}\}$ are labeled by
$z^*(x_i x_{i+2}) = \left\lfloor \frac{z(x)z(y)\{z(x)+z(y)\}}{z^2(x)+z^2(y)} \right\rfloor$,
for every $\{x_i, x_{i+2}\} = e \in E(P_n^2)$, are distinct,
As a result, $z^*$is injective and P_n^2 is an invariant contra harmonic graph labeling.
Illustration: The Invariant contra harmonic mean labeling of **Square graph** $P_7^2$is as shown in the following figure.



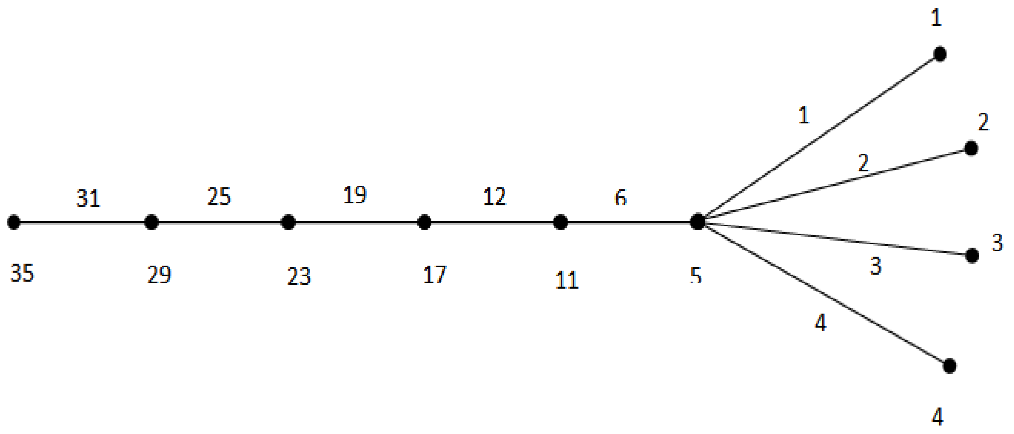
Theorem 2.3. The broom graph is an invariant contra harmonic graph labeling.
Proof: The broom graph with $n+m$ vertices and $n+m-1$ edges.
Let u be vertex of P_n and $v_1, v_2, \ldots., v_m$ be the pendent vertices incident on one end of the path.
The ordinary labeling of $v_1, v_2, \ldots., v_m$ is a natural numbers.
Define a vertex labeling for path P_n $z: V(P_n) \to 5,11,17, \ldots., n$ by $z(x) = 6n-1$; for $n \ge 1$.
The pendent edges are labeled by the invariant contra harmonic,

$$z^*(v_i u_1) = \left\lfloor \frac{z(v_i)z(u_1)\{z(v_i)+z(u_1)\}}{z^2(v_i)+z^2(u_1)} \right\rfloor$$

For every $v_i, u_1 \in V(G)$.
The edges of the path are labeled by the invariant contra harmonic,

$$z^*(u_i u_{i+1}) = \left\lfloor \frac{z(u_i)z(u_{i+1})\{z(u_i)+z(u_{i+1})\}}{z^2(u_i)+z^2(u_{i+1})} \right\rfloor$$

For every $u_i, u_{i+1} \in V(G)$ and $u_i \ne u_{i+1}$.
are distinct. Thus, the broom graph is an invariant contra harmonic graph labeling.
Illustration: An invariant contra harmonic graph labeling of broom graph is as shown in the following figure.



Theorem 2.4. Comb an invariant contra harmonic graph labeling.
Proof: Take G be a comb. Assume P_n be the path $v_1, v_2, v_3 \cdots v_n$.
Define a function $z: V(G) \to \{1, 2, \ldots, p\}$ by
For $1 \leq i \leq n$,

$z(u_i) = 2i$, $f(u_i) = 2i$

and $z(v_i) = 2i - 1$,

The label of the edge, $\{u_i v_j\}$ is

(i) For $1 \leq i \leq n$,

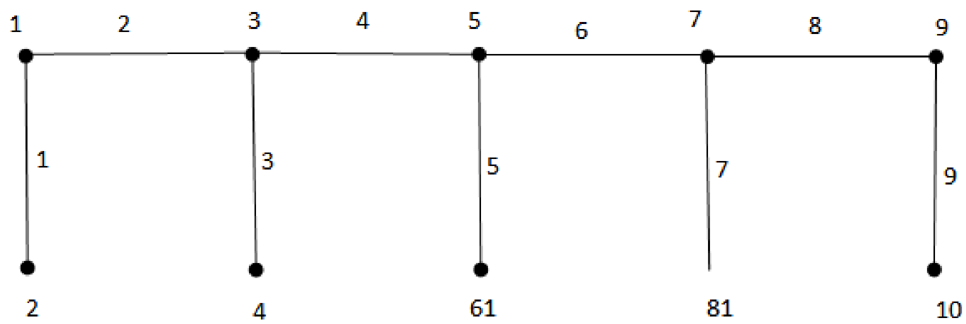
$$z^*(u_i v_i) = \left\lceil \frac{z(u_i)z(u_{i+1})\{z(u_i) + z(u_{i+1})\}}{z^2(u_i) + z^2(u_{i+1})} \right\rceil = 2i - 1,$$

for all $u_i, v_i \in V(G)$ and $u_i \neq v_i$.
The label of the edge $\{v_i v_{i+1}\}$ is

(ii) For $1 \leq i \leq n$,

$$z^*(v_i v_{i+1}) = \left\lceil \frac{z(v_i)z(v_{i+1})\{z(v_i) + z(v_{i+1})\}}{z^2(v_i) + z^2(v_{i+1})} \right\rceil = 2i,$$

for all $v_i \in V(G)$ and $v_i \neq v_{i+1}$, are all distinct.
Hence combs are an invariant contra harmonic graph labeling.
Illustration: An invariant contra harmonic graph labeling of comb graph is as shown in the following figure.



Theorem 2.5. A Ladder L_n is an Invariant contra harmonic mean labeling graph.
Proof: Consider the Ladder L_n. Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ be two n-length paths in the ladder L_n.
Create a function $z: V(TL_n) \to \{1, 2, \ldots, p + q\}$ by,
For $1 \leq i \leq n$, $z(x_i) = 3i - 1$; and $z(y_i) = 3i - 2$;
Edges are labeled by,
(i) For $1 \leq i \leq (n - 1)$,

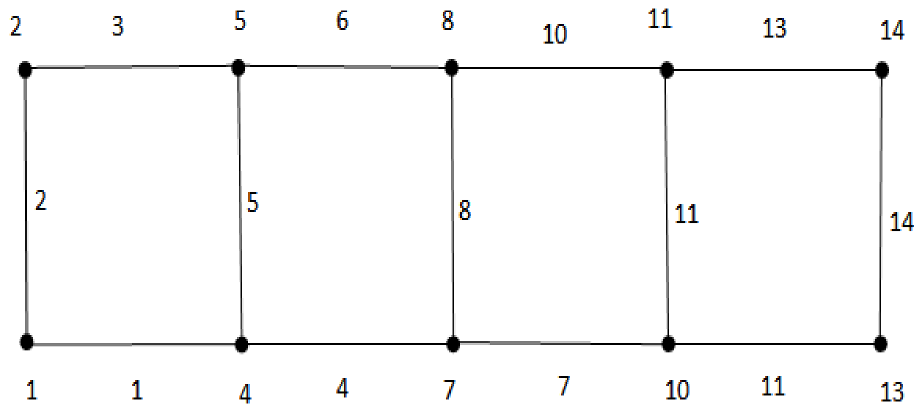
$$z^*(x_i x_{i+1}) = \left\lceil \frac{z(x_i)z(x_{i+1})\{z(x_i) + z(x_{i+1})\}}{z^2(x_i) + z^2(x_{i+1})} \right\rceil;$$
$$z^*(y_i y_{i+1}) = \left\lceil \frac{z(y_i)z(y_{i+1})\{z(y_i) + z(y_{i+1})\}}{z^2(y_i) + z^2(y_{i+1})} \right\rceil;$$

(ii) For $1 \leq i \leq n$,

$z^*(x_i y_i) = \left\lceil \frac{z(x_i)z(y_i)\{z(x_i)+z(y_i)\}}{z^2(x_i)+z^2(y_i)} \right\rceil$; are all distinct.

We recognise thatz^* is an ICHM characterization and that the ladder L_n is ICHM graphs.

Illustration: An ICHM graph labeling of Ladder L_n with vertices is as shown in the following figure.



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